MATH-905 Symmetry Methods for Differential Equations-I

Credit Hours: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Symmetry Methods. Whereas there are standard techniques for solving differential equations, apart from the first order equations there are no standard techniques for solving non-linear differential equations. Lie had developed an approach to try to determine substitutions, whichcould be used to reduce the order of an ODE, or the number of independent variables of a PDE. This field has made dramatic advances under the name of "symmetry analysis". In this course Lie groups, local Lie groups and Lie algebras will be reviewed. Then the symmetries of algebraic and differential equations will be discussed. Next the techniques for finding the symmetries of an ODE, and their use for solving it will be presented. This will be extended tosystems of ODEs. The technique of finding differential invariants will be discussed with reference to some particular examples. The symmetries of PDEs will also be discussed and some examples presented.

Core Contents: Lie groups, local Lie groups and Lie algebras. Symmetries of algebraic and differential equations. Techniques for finding the symmetries of an ODE and their use for solving it. Extension to systems of ODEs. Differential invariants. The symmetries of PDEs. Techniques for finding the symmetries of a PDE, and their use for reducing the number of independent variables.

Detailed Course Contents: One-parameter group of point transformations and their generators, Transformation laws, Extensions of transformations. Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1^{st} order, Second formulation of symmetries Lie symmetries of 1^{st} and 2^{nd} order ODEs. Lie symmetries of 2^{nd} order ODEs; higher order ODEs and linear n^{th} order ODEs. The use of symmetries to solve 1^{st} order ODEs. Lie algebras for infinitesimal generators. Examples of Lie Algebras. Subgroups and subalgabras; Invariants and Differential Invariants. The use of symmetries for solving 2^{nd} order ODEs admitting a G_2. Second integration strategy. The use of symmetries for solving 2^{nd} order ODEs admitting more than two symmetries. Higher order ODES admitting more than one Lie point symmetry. System of second order differential equations. Symmetries more general than Lie point symmetries. Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1^{st} order. 2^{nd} order PDEs; Generating solutions by Symmetry transformations.

Course Outcomes: On successful completion of this course, students will be able to

- understand the basic concepts of the Lie point symmetries
- determine the symmetries of differential equations
- use symmetries to get the solutions or reduce order of ordinary differential equations
- determine the symmetries of system of ordinary differential equations

- determine the Noether symmetries of differential equations
- understand the need of contact symmetries of differential equations

Textbook: Hans Stephani, Differential Equations: Their Solution Using Symmetries, Cambridge University Press 1990

Reference book: N. H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley and Sons 1999.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.3	One-parameter group of point transformations and their generators, Transformation laws, Extensions of transformations.
2	2.4, 3.1-3.2	Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1^{st} order
3	3.3-3.4, 4.1-4.2	Second formulation of symmetries Lie symmetries of 1^{st} and 2^{nd} order ODEs.
4	4.3, 4.4	Lie symmetries of 2^{nd} order ODEs; higher order ODEs and linear n^{th} order ODEs.
5	5.1-5.2	The use of symmetries to solve 1^{st} order ODEs.
6	6.1-6.2	Lie algebras for infinitesimal generators. Examples of Lie Algebras.
7	6.3-6.5	Subgroups and subalgabras; Invariants and Differential Invariants.
8	7.1-7.2	The use of symmetries for solving 2^{nd} order ODEs admitting a G_2.
9	**Mid Semester Exam**	
10	7.3-7.4	Second integration strategy.
11	7.5, 8.1-8.3	The use of symmetries for solving 2^{nd} order ODEs admitting more than two symmetries.
12	9.1-9.5	Higher order ODES admitting more than one Lie point symmetry.
13	10.1-10.3	System of second order differential equations.
14	11.1-11.5	Symmetries more general than Lie point symmetries.
15	15.1-15.3 16.1	Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1^{st} order.
16	16.2, 17.1-17.4	2^{nd} order PDEs; Generating solutions by Symmetry transformations.
17		Review
18	**End Semester Exam**	